P425/1
PURE
MATHEMATICS
Paper 1
AUGUST 2018
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS AUGUST 2018

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in section A and any **five** from section B.*
- *Any additional question (s) answered will not be marked*
- *All necessary working **must** be shown clearly*
- *Begin each answer on a fresh sheet of paper*
- *Graph paper is provided*
- *Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

*Answer **all** questions in this section*

1. Find the coefficient of the term involving x^{-6} in the expansion of $\left(3x-\frac{2}{x^2}\right)^9$. (05marks)

2. Use the identities $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find the value of $\sin 18^o$. Leave your answer in surd form. (05marks)

3. If the function $f(x) = x^3 + ax^2 + bx + c$ has stationary values at $x = \alpha$ and $x = \beta$. Show that $\frac{f(\alpha)-f(\beta)}{\alpha-\beta} = \frac{2}{9}(3b - a^2)$ (05marks)

4. Given that the angle between the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \lambda\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ is $\cos^{-1}\left(\frac{1}{2}\right)$, find the value of λ. (05marks)

5. Evaluate $\int_0^1 e^{\sqrt{x}}\,dx$. (05marks)

6. Prove by mathematical induction that $3^{2n+1} + 5^{2n-1}$ is divisible by 16, for $n \geq 1$. (05marks)

7. Find the equations of the lines that bisect the angles between the pairs of lines $3x = 4y + 2,\ 12y = 5x + 2$. (05marks)

8. Solve the differential equation:
$(x+1)\frac{dy}{dx} - 3y = (x+1)^4$, given that $y(1) = 6$. (05marks)

SECTION B: (60MARKS)

*Answer any **five** questions from this section. All questions carry equal marks*

9. (a) Use Demoivre's Theorem to show that; $\tan 3\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$. (05marks)

(b) Find the roots z_1 and z_2 of the equation, $(Z+i)^2 = 3-4i$. Hence represent the roots on the Argand diagram. (07marks)

10. (a) Solve the equation $2(\sin x + \sin 2x) - 1 = \cos 2x + \cos x$, for $0^\circ \le x \le 360^\circ$. (04marks)

(b) Express $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ in the form $q + r\cos(2x - \alpha)$, where q, r are constants and α is an acute angle. Hence find the maximum and minimum values of $\dfrac{9}{5\sin^2 x - 3\sin x\cos x + \cos^2 x}$. (08marks)

11. Given that $\dfrac{x^2 - 8x + 5}{(2x+1)(x^2+9)} = \dfrac{A}{2x+1} + \dfrac{Bx+c}{x^2+9}$, find the values of A, B and C.

Hence show that $\int_0^3 \dfrac{x^2 - 8x + 5}{(2x+1)(x^2+9)}\,dx = \dfrac{1}{2}In7 - \dfrac{\Pi}{3}$. (12marks)

12. The polynomial $f(x) = x^4 + 4x^3 + lx^2 + mx + n$ is a perfect square of second degree.

(a) Show that $m + 8 = 2l$ and $16n = m^2$. (06marks)

(b) If the polynomial $f(x)$ leaves a remainder 4 when divided $x+1$. Determine the possible values of l, m and n. (06marks)

13. The parametric equations of two planes Π_1 and Π_2 are;

$$\mathbf{r} = \begin{pmatrix}2\\4\\3\end{pmatrix} + \alpha\begin{pmatrix}-1\\0\\2\end{pmatrix} + \beta\begin{pmatrix}1\\2\\-8\end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix}-2\\-3\\0\end{pmatrix} + \mu\begin{pmatrix}-1\\-1\\-1\end{pmatrix} + \lambda\begin{pmatrix}-3\\-4\\-2\end{pmatrix} \text{ respectively.}$$

(a) Find the cartesian equation of each plane. (06marks)

(b) If l is the line of intersection of the planes, find the;

(i) equation of the line, l ,in vector form. (03marks)

(ii) coordinates of the foot of the perpendicular from the point $P(1,-5,-10)$ to the line, l . (03marks)

14 (a) Given that $y=\sqrt{1-\cos^2 x^2}$. Find the value of $\frac{dy}{dx}$ at $x=\frac{\sqrt{\Pi}}{2}$. (06marks)

(b) If $y=e^{4x}\cos 3x$, prove that $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+25y=0$. (06marks)

15 (a) Find the equations of the circles passing through the points of intersection of the circles $x^2+y^2-18x-2y+8=0$ and $x^2+y^2-26x+6y=24$ and touching the line $y=10$. (05marks)

(b) P and Q are points $(ap^2,2ap)$ and $(aq^2,2aq)$ respectively on the parabola $y^2=4ax$. If the tangents to the parabola at the given points intersect at R and are inclined at angle of 45^o. Show that the locus of R is the curve, $y^2-x^2-6ax-a^2=0$. (07marks)

16. Army worms wipe out a community at a rate proportional to the population present at any time. If the initial population is 4million and it reduces from 2.5million to one-fifth of a million in 5months. How long does it take the initial population to reduce to a million? (12marks)

 End.

P425/2
APPLIED
MATHEMATICS
Paper 2
AUGUST 2018
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

AUGUST 2018

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question (s) answered will not be marked*
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- *Begin each answer on a fresh sheet of paper*
- *Graph paper is provided*
- *Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*
- *In numerical work, take g to be 9.8 ms^{-2}.*

SECTION A (40 MARKS)

1. A student may travel to school by car, by bicycle or on foot. The probability of using a car is $\frac{1}{3}$, of riding a bicycle is $\frac{1}{2}$ and of going on foot is $\frac{1}{6}$, when he uses a car, the probability of arriving in time is $\frac{3}{4}$, by bicycle is $\frac{2}{5}$, on foot is $\frac{1}{10}$.

   Calculate the probability that the student went on foot given that he did not arrive in time. (05marks)

2. An object performs Simple Harmonic Motion (S.H.M) at rate of 20 oscillations per second between two points **A** and **B** which are 12cm. If **C** is the midpoint of **AB**, calculate the time taken to travel directly from **C** to the midpoint of **CB**. (05marks)

3. Given the numbers $x = 15.2$, $y = 0.45$ and $z = 8.6$, all measured to the nearest number of decimal places as indicated.
   (i) State the maximum possible errors in each of the numbers. (01mark)
   (ii) Find the percentage error in $\frac{xy}{z}$. (04marks)

4. Given that $A \sim N(28.9)$. Find the $P(22 < A < 26)$. (05marks)

5. The diagram in figure 1 below shows a body which may be modelled as a uniform lamina.

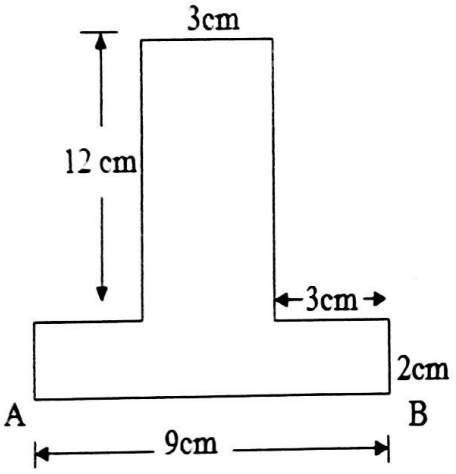

Figure 1

If the body is suspended from the point marked A and rests in equilibrium, calculate the angle which the edge AB makes with the vertical. (05marks)

6. The sizes of shoes sold in a certain shop in a given week are shown in table 1 below.

Size	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0
Number of pairs of shoes sold.	4	9	11	8	10	7	2	3

Find the:

(i) Mean. (02marks)

(ii) Standard deviation of the sizes of shoes sold. (03marks)

7. Table 2 below shows the values of x and the corresponding values of $f(x)$.

x	50.24	48.11	46.93	44.06
$f(x)$	4.116	7.621	9.043	11.163

Use linear interpolation/extrapolation to find the value of

(i) x when $f(x) = 8.614$. (03marks)

(ii) $f(51.07)$. (02marks)

8. A particle projected from a point O at angle of $50°$ above the horizontal passed through the point P, with position vector $70\mathbf{i} + 28\mathbf{j}$. Find the

(i) initial velocity (03marks)

(ii) time taken to reach P. (02marks)

SECTION B (60MARKS)

*Answer any **five** questions from this section. All questions carry equal marks*

9. The probability density function of a continuous random variable X is given as

$$f(x) = \begin{cases} \frac{2}{13}(x+1); 0 \le x \le 2 \\ \frac{2}{13}(5-x); 2 \le x \le 3 \\ 0; \ otherwise \end{cases}$$

(a) Calculate the

(i) $P(X < 2.5)$.

(03marks)

(ii) Mean of X. (03marks)

(b) Determine the cumulative distribution function, $F(x)$. (06marks)

10. (a) A block of mass $12kg$ is placed on a rough plane, inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is 0.5. If the block is kept in equilibrium by the horizontal force of magnitude P Newtons, find the range of possible values of P. (06marks)

(b) A non uniform rod AB of mass $20kg$ and length $4m$ is suspended horizontally from the ends of strings AC and BD such that AC and BD make an angle of 60° and 45° respectively with the vertical. If the tension in AC is $60N$,

Calculate the

(i) tension in string BD. (03marks)

(ii) distance from A where the weight of the rods acts. (03marks)

11. (a) Show that the equation $xe^{x} = x+1$, has a root between 1 and 2. (3marks)

(b) Use the Newton Raphson Method to find the root of the equation in (a) above correct to 2 decimal places. (09marks)

12. (a) The chance of a goat being infected on a farm is 0.4. If there are 150 goats on the farm, find the probability that

(i) more than 55 goats will be infected. (04marks)

(ii) between 60 and 70 goats are infected. (04marks)

(b) The heights of students in a certain school are normally distributed with mean 164cm and standard deviation 7.2cm.
Calculate the probability that the mean height of a sample of 40 students will be between 162 and 166cm. (04marks)

13. The diagram in figure 2 below shows the velocities and initial positions of particles A and B.

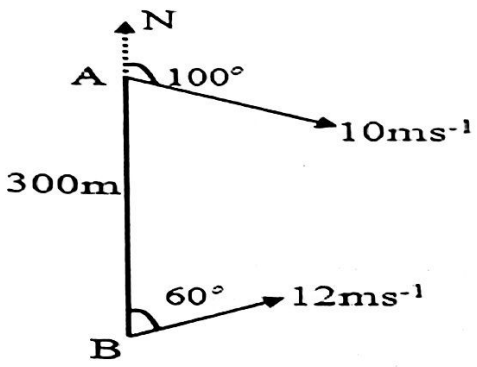

Figure 2

Calculate the

(a) shortest distance between the two particles. (08marks)

(b) time taken to reach that position. (04marks)

14. (a) Use the trapezium rule with six ordinates to estimate

$$\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1+\cos x}}\,dx,$$

correct to three decimal places. (06marks)

(b) Study the flow chart in figure 3 below and answer the questions that follow

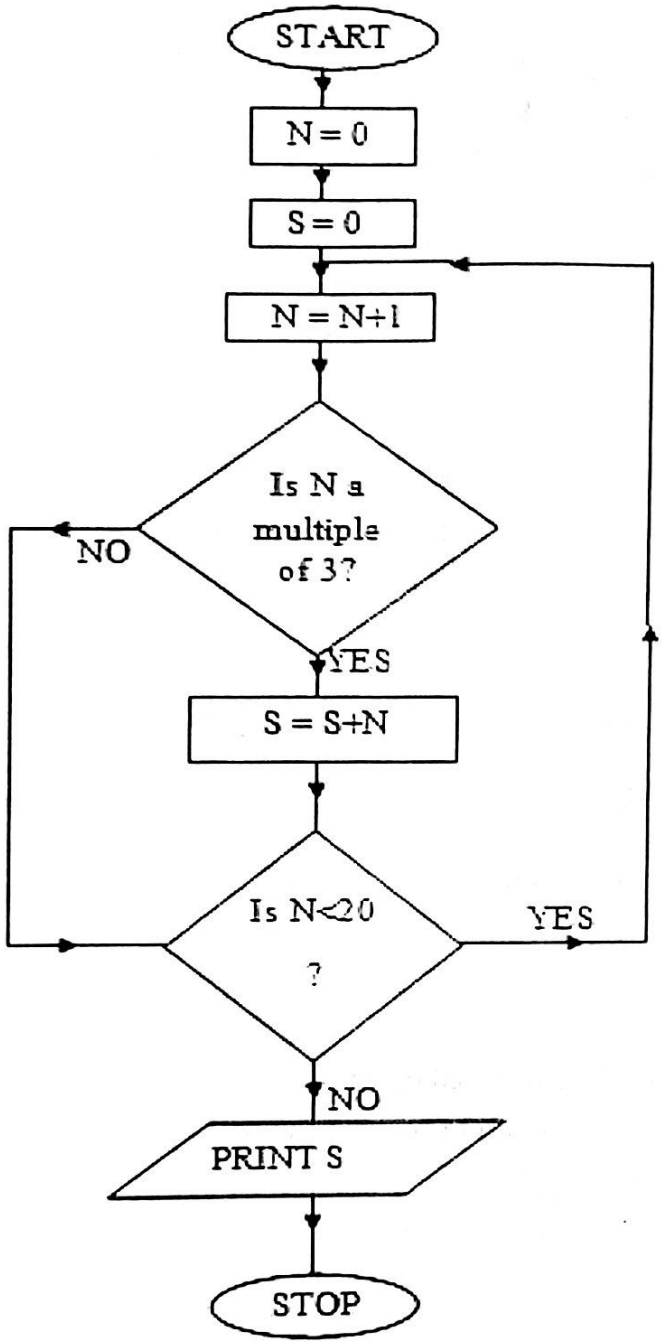

Figure 3

(i) Perform a dry run for the flow chart.

(ii) State the purpose of the flow chart. (06marks)

15. A company took a survey of the ages of its employees. The results are shown in the frequency table below.

Ages (yrs)	18 –	20 –	30 –	40 –	50 –
frequency	8	22	15	7	2

(a) Calculate the

(i) mean age. (03mark)

(ii) modal age. (03marks)

(iii) standard deviation. (02marks)

(b) Draw a cumulative frequency curve and use it to estimate the middle 60% age range. (04marks)

16. (a) A bus of mass 18tonnes travels up a slope inclined at $\sin^{-1}\left(\frac{1}{50}\right)$ against a resistance of $0.1N$ per kilogram. Find the tractive force required to produce an acceleration of $0.05ms^{-2}$ and the power which is developed when the speed is $10ms^{-1}$. (06marks)

(b) A second bus of mass 25tonnes experiencing the same resistance and with a maximum power of $120KW$ follows the first bus up the slope. If the first bus maintains the same power while on the slope, find the maximum speed of the second bus. (06marks)

 End.